(1) A fixed instrument for observation of *Polaris*, by means of which continuous observation of high accuracy would be possible.

(2) A transit instrument in a plane through the pole at right angles to the meridian.

The different historical values of the precession are summarised.

The values which have been attributed to the proper motion of *Polaris* in connection with the different precessions are exhibited in a comparable form in *Polaris* coordinates.

Places of the pole of the ecliptic and of the pole are given in *Polaris* coordinates for different dates; also a simple approximate method of dealing with the motion of the pole.

Suggestions are made for the tabulation of the functions which connect two systems of spherical coordinates, whose poles are not far apart, both in connection with the application of the above system to particular stars and as a more general question of spherical geometry.

Note.—The above is a summary of the chief contents of a paper, bearing the same title, which is to be placed in the library of the Society, together with a supplementary paper entitled "On the Star Survey by Fixed Coordinates in the Southern Hemisphere; with a Proposal for Cooperation."

On the Relation between Precession and Proper Motion.
By R. H. M. Bosanquet, F.R.S.

My former paper on a fixed system of star coordinates was based to some extent on the idea that, as changes in the adopted precession had been found desirable in the past, the same might happen in the future. I therefore saw with interest Mr. Stone's paper on the places of Eight Close Southern Polar Stars (*Monthly Notices*, LV., 299), where the argument is that the precession must be right because the residual proper motions are so small.

Now I have for some time been aware that the proper motion of *Polaris* is susceptible of diminution by an increase of the adopted precession; and it occurred to me in examining *σ Octantis*, as a possible pole of a southern star survey, to discuss the proper motion from this point of view. The result is that the proper motion of *σ Octantis* is also susceptible of diminution by an increase of the adopted precession; and the values of the increase of precession required to make the proper motion a minimum are similar in the two cases.

On the other hand 51 *Cephei* (Hev) requires a diminution of the precession, somewhat greater than the increase of the former cases, to make the proper motion a minimum.

These three stars form a system determining, by the combination of their minimum requirements, a precession differing not

very widely from that in use. And I propose to use them as illustrations in support of two propositions.

First, in the separate cases, certain large definite alterations in the precession cause no change in the magnitude of the resultant proper motions, while smaller alterations even diminish them.

Second, where the precession is considered with reference to a number of stars whose minimum proper motions require different precessions, large changes in the precession may produce very small changes in the average magnitude of the resultant proper motions within wide limits.

If these propositions are established, I think that no argument can be founded on the smallness of the proper motions to prove the precise exactness of the precession in use.

Every star has a minimum value for its proper motion, which occurs when the direction of the proper motion is along the great circle joining the star to the pole of the ecliptic. If the proper motion have any other direction, let it be resolved along, and at right angles to that great circle. Then a value of the precession can always be found, which shall absorb the component at right angles to the great circle, leaving the component along the circle as an irreducible minimum.

The process pursued in evaluating these quantities may be sketched as follows:—

A_ϵ is the angle at the star between any origin and the pole of the ecliptic.

A the angle between the same origin and the direction of the proper motion.

$A_\epsilon - A$ the inclination of the proper motion to the great circle through the star and the pole of the ecliptic.

$d\Sigma''$ is the resultant proper motion.

$d\Sigma'' \cos (A_\epsilon - A)$ component along circle to pole of ecliptic.

$d\Sigma'' \sin (A_\epsilon - A)$ component at right angles to circle.

D is the angular distance from the star to the pole of the ecliptic.

$\dfrac{d\Sigma'' \sin (A_\epsilon - A)}{\sin D}$ is the change of precession required for minimum proper motion.

$d\Sigma'' \cos (A_\epsilon - A)$ is the minimum proper motion.

In the case of *Polaris* the required data are taken from my paper "On a Fixed System of Star Coordinates," except the proper motion. I have reason to suppose, after examining Auwers' Bradley, that the discrepancy between the proper motion there found by comparison and the modern values, may be due chiefly

to the errors of the places of the Fundamenta. I therefore adopt the mean of the following values :—

Polaris.	$d\Sigma''$.	A of pole of motion, about Polaris, from ϵ Orionis.
	$''$	° ′ ″
Auwers	0·0393	294 54 0
Greenwich, 1880 ...	0·0387	293 13 0
Radcliffe, 1890... ...	0·0381	293 59 0
Mean	0·0387	294 2 0
For direction of motion add		90 0 0
A from ϵ Orionis		24 2 0
A_ϵ from ϵ Orionis		184 38 0
D „		23 54 39

For the other stars a system of calculation was arranged.

Pole of the ecliptic.

A_ϵ, reckoned from the R.A. plane of the star, and D were found by formulæ of the same types as transformations I, II in my former paper.

Proper motion.

A, reckoned from the R.A. plane of the star, and $d\Sigma''$ were found as follows :—

$$(1)\quad \tan A = \frac{\sin \Delta d\alpha''}{d\Delta''} \qquad\qquad (2)\quad d\Sigma'' = \frac{d\Delta''}{\cos A}.$$

In the case of σ *Octantis* there was a slight difficulty. The proper motion assigned by Mr. Stone (*Monthly Notices*), lv. 299, is not there referred to any definite equinox. Assuming that the *Nautical Almanac* place for 1895 is correct (it is probably founded on Mr. Stone's calculations), I find the following proper motions for 1880 and 1895, through the medium of the *Polaris* places given in my paper on the "Star Survey by Fixed Coordinates in the Southern Hemisphere."* I now compare these proper motions with Mr. Stone's from the above reference :—

σ Octantis.	R.A.	N.P.D.
	$''$	$''$
1880	+1·049	+0·022
1895	+1·216	+0·020
Stone	+1·125	+0·02

* The *Polaris* proper motion was allowed for with the value given above, as well as the proper motion of ϵ *Orionis* in ascension, which is $-0''\cdot027$ annually. Then the resultant of the remaining proper motion of the star was formed. The parallactic angle at the star between the *Polaris* and R.A. circles was found for the two dates, and the resultant proper motion resolved accordingly.

I used the numbers derived from the 1880–1895 comparison, taking everything with reference to 1880.

The proper motion and place of 51 *Cephei* (Hev.) were taken from the Greenwich Catalogue, 1880.

The value of ω employed for 1880 was

$$23^\circ\ 27'\ 17''\cdot40.$$

The following numbers were found—

	σ Octantis.	51 Cephei (Hev.).
	° ′ ″	° ′ ″
A_ϵ	174 3 0	170 20 44
A	148 54 0	− 29 37 40
D	24 10 34	26 46 16
$d\Sigma''$	0 0 0·0257	0 0 0·0586

The following table exhibits the results (SE is the great circle from the star to the pole of the ecliptic) :—

Resolution of Proper Motions along and at Right Angles to SE; and Changes involved by Reduction of Proper Motion to Minimum.

	Resultant Proper Motion.	$A_\epsilon - A$.	Minimum Component along SE.	Numerical Diminution of Resultant.	Change in direction.	Component Perpendicular to SE.	Change of Precession required for diminution to minimum.
	″	° ′	″	″	° ′	″	″
Polaris	0·0387	160 36	−0·0365	0·0022	−19 4	+0·0129	+0·0317
σ Octantis	0·0257	25 9	+0·0233	0·0024	+25 9	+0·0109	+0·0267
51 Cephei (Hev.)	0·0586	199 58	−0·0551	0·0035	+19 58	−0·0200	−0·0445
						Mean	+0·0046

Resultants for Changes of Precession.

Change of Precession.		+0″·0317.	−0″·0445.
		″	″
	Polaris	0·0365	0·0478
	σ Octantis	0·0233	0·0373
	51 Cephei	0·0650	0·0551
Average of resultants	0″·0410	0·0416	0·0467

The reduction to the minimum changes the direction of the motion through the angle, "change in direction."

The application of twice the "change of precession required, &c.," would bring back the numerical resultants to their original values; but they would be turned through twice the angle, "change in direction."

The "change of precession required, &c.," is, for *Polaris* and *σ Octantis*, about twice the difference between Bessel's last

number (III. of my paper "On a Fixed System, &c.") and the modern value (Struve). It is about three times that difference for 51 *Cephei*.

Therefore, in the cases of *Polaris* and σ *Octantis*, the resultant, after diminishing, returns to the same numerical value, when the precession is increased by four times the difference (Struve—Bessel, III.). In the case of 51 *Cephei* the diminution of the precession, after which the resultant returns to its original value, is about six times that difference.

As far as the separate stars go, therefore, my first proposition is amply proved, viz., that, in the separate cases, large definite alterations in the precession produce no change in the magnitude of the resultant proper motions, while smaller alterations even diminish them.

We now pass to the average magnitude of the resultant proper motions: this is given for the precession in use and for the precessions corresponding to the minimum proper motions of *Polaris* and 51 *Cephei*. This covers a range of five times the difference (Struve—Bessel III.), and the changes in the average magnitude of the resultants are very small.

There would, in a general determination, be cases of stars having their proper motions nearly at right angles to SE. In these cases only would a change of precession determine directly a commensurate change in the resultant proper motion.

I think I may now claim that my second proposition is established; that, when the precession is considered with reference to a number of stars whose minimum proper motions require different precessions, large changes in the precession may produce very small changes in the average magnitude of the resultant proper motions within wide limits; and, consequently, no argument can be founded on the smallness of the proper motions to prove the precise exactness of the precession in use.

Incidentally it appears that the best method of determining the precession from the proper motions of stars is the combination of the various precessions which determine the minimum proper motions of the stars concerned.

It is worth remark that circumpolar stars and other stars far from the ecliptic are unsuitable for the determination of precession, since the component at right angles to SE, by means of which the change of precession is found, is, in such cases, much less than the change of precession itself. Stars near the ecliptic are most suitable, as in these cases the component in question is nearly equal to the change of precession it determines.

Teneriffe: 1895 *May* 21.

Ephemeris for Physical Observations

Greenwich Noon.		Position-angle of ♃'s Axis. P	L−O	Diff.	B	Annual Parallax. Λ−L	Apparent Diameter. Equat. 2a	Defect.	Polar. 2b
1895.		°	°		°	°	"	"	"
Aug.	30	13·598	343·587	394	+1·051	−6·796	33·43	0·12	31·33
Sept.	1	13·750	343·981	389	1·035	7·026	33·53	·13	31·43
	3	13·900	344·370	384	+1·019	−7·251	33·64	0·13	31·53
	5	14·047	344·754	380	1·003	7·472	33·76	·14	31·64
	7	14·192	345·134	375	0·988	7·688	33·88	·15	31·75
	9	14·334	345·509	369	·972	7·899	34·00	·16	31·87
	11	14·473	345·878	364	·957	8·104	34·13	·17	31·99
	13	14·609	346·242	358	+0·941	−8·304	34·27	0·18	32·12
	15	14·743	346·600	352	·925	8·498	34·41	·19	32·25
	17	14·874	346·952	346	·910	8·687	34·55	·20	32·38
	19	15·002	347·298	339	·894	8·870	34·70	·21	32·52
	21	15·127	347·637	333	·879	9·046	34·85	·22	32·66
	23	15·249	347·970	326	+0·864	−9·215	35·01	0·23	32·81
	25	15·367	348·296	319	·849	9·378	35·17	·23	32·96
	27	15·483	348·615	312	·834	9·534	35·33	·24	33·11
	29	15·596	348·927	305	·819	9·683	35·50	·25	33·27
Oct.	1	15·705	349·232	297	·805	9·824	35·67	·26	33·44
	3	15·811	349·529	288	+0·790	−9·957	35·85	0·27	33·61
	5	15·914	349·817	280	·775	10·082	36·04	·28	33·78
	7	16·014	350 097	272	·761	10·200	36·22	·29	33·95
	9	16·110	350·369	263	·747	10·309	36·41	·29	34·13
	11	16·202	350·632	255	·733	10·409	36·61	·30	34·31
	13	16·291	350·887	245	+0·720	−10·500	36·81	0 31	34·50
	15	16·376	351·132	236	.706	·583	37·01	·31	34·69
	17	16·458	351·368	226	·693	·656	37·22	·32	34·89
	19	16·536	351·594	216	·680	·719	37·43	·33	35·09
	21	16·611	351·810	205	·668	·772	37·65	·33	35·29
	23	16·682	352·015	196	+0·655	−10·815	37·87	0·34	35·49
	25	16·749	352·211	185	·643	·848	38·09	·34	35·70
	27	16·812	352·396	174	·631	·870	38·32	·34	35·91
	29	16·872	352·570	162	·619	·881	38·55	·35	36·13
	31	16·927	352·732	151	·608	·881	38·78	·35	36·34
Nov.	2	16·979	352·883	140	+0·597	−10·870	39·01	0·35	36·56